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Plants Versus ZZombies: Do Plants Respond Differently to Vivaldi or Metal? - Acoustic Input as an Agricultural Production Factor: A Greenhouse Pre-Analysis Plan with Potential Applications for Poverty Alleviation and Sustainability Funding

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Abstract
Global hunger remains structurally persistent, and current trajectories imply that nearly 600 million people will still face hunger in 2030 [1,2]. Agricultural productivity growth is increasingly constrained by nonlinear heat stress, groundwater depletion, CO_2-induced reductions in crop micronutrient concentrations, and increased climate variability and extremes [3-6]. This proposal evaluates whether controlled acoustic stimulation could operate as a candidate marginal agricultural input. While plant mechano-biology and related agronomic studies report measurable physiological and growth responses to sound exposure that are often heterogeneous across species, frequencies, and contexts, economic integration, causal validation, and structural cost–benefit modelling remain missing [7-10]. One can embed acoustic input within a Cobb–Douglas production framework, extend it to stochastic climate environments, derive dynamic adoption conditions under risk aversion, and design a power-randomized controlled trial (RCT) linked to structural calibration. The objective for the proposed future research is a disciplined empirical test of a potentially scalable productivity margin under food security SDG II.

Keywords: Food Security, Agricultural Productivity, Poverty Alleviation, Causal Inference, Production Theory
JEL Codes: Q10; Q18; C31; O13; I32

Introduction
Despite sustained global economic growth, global hunger remains persistent. More than 600 million individuals remain undernourished [1]. Progress toward Sustainable Development Goal (SDG) II has slowed significantly [2].

Agricultural systems face compounding structural pressures in a VUCA world with multiple crises. Rising temperatures cause nonlinear crop yield effects [3]. Groundwater extraction exceeds recharge in key basins [4]. Elevated CO_2 concentrations are associated with declining micronutrient density in staple crops [5]. Climatic variability, such as shocks, including aerosol masking and volcanic cooling, introduces persistent uncertainty [11,12].

Traditional intensification margins, land expansion, irrigation scaling, and fertilizer intensification are ecologically constrained. This motivates systematic evaluation of unconventional but very testable productivity margins, especially with the use of the modern "Big Five" Causal Inference Toolkit.

We evaluate whether acoustic stimulation can be treated as a marginal production factor within a formal economic framework.

Agricultural Production Framework

From mechanoperception to a production input

Plants are known to be mechanosensitive, meaning that mechanical perturbations (touch, wind, vibration) are transduced into biochemical signals (e.g., via mechanosensitive ion channels, cytoskeletal remodelling, and downstream gene regulation), producing measurable phenotypic responses, as often discussed in the literature on thigmomorphogenesis and plant mechanobiology [13-16]. Sound exposure can be viewed as a controlled delivery of mechanical energy through pressure waves that induce tissue-level vibrations; empirical studies report heterogeneous effects on germination, growth, and secondary metabolism, with responses depending on frequency, intensity, duration, and species [7,17,18].

Motivated by this mechanistic plausibility, one can treat controlled acoustic stimulation as a candidate marginal input that can shift growth rates and/or the efficiency with which conventional inputs translate into biomass and yield.

Microfoundation: Growth Dynamics Under Acoustic Dose

Let $B(t)$ denote plant biomass (or a relevant growth state) over $t \in [0, T]$. Biomass accumulation follows

$$\grave{B}(t) = \phi(L(t), K(t), W(t), N(t); \omega) \cdot g(D(t)) \cdot \exp(\varepsilon(t)) \tag{1}$$

where L, K, W, N are labor, capital, water, and nutrients (or fertilizer) applied over time; ω collects fixed biological/management traits (genotype, cultivar, soil class, baseline technology); $g(\cdot)$ captures the net growth modulation from acoustic exposure; and $\varepsilon(t)$ is an environmental shock process (weather realizations, insects, weeds, or plant diseases, measurement noise).

An Acoustic dose. Define an acoustic dose $D(t)$ that aggregates treatment intensity and exposure duration. A minimal formulation is

$$D(t) = \int_{t-\Delta}^{t} I(\tau) d\tau \tag{2}$$

where $I(\tau)$ is the sound intensity at the canopy level (e.g., in W/m^2 or mapped from dB SPL) and Δ is the relevant biological integration window. More generally, $D(t)$ can be frequency-weighted to reflect that plant responses are frequency dependent:

$$D(t) = \iint w(f)I(f,\tau)df\, d\tau \quad (3)$$

with $w(f)$ an unknown weighting function to be estimated or (e.g., from agronomic pilot experiments).

Functional restrictions. One can assume $g(0) = 1,\ g'(D) \geq 0$ locally (allowing nonmonotonicity outside the experimental range) and diminishing returns $g''(D) \leq 0$ over the policy-relevant domain. Heterogeneity is expected across crops and environments [17,18].

Integrating (1) over the growing cycle yields terminal biomass $B(T)$ and harvest-able output Q as a function of cumulative inputs and realized shocks. Under standard regularity conditions (small fluctuations around a baseline, smooth responses), a log-linear approximation motivates an aggregate production representation in which the acoustic dose enters as either (i) a Hicksianneutral shifter or (ii) an additional factor:

$$Q = AL^{\alpha}K^{\beta}W^{\gamma}N^{\delta}S^{\theta} \cdot \exp(u) \quad (4)$$

where S is a season-level summary of dose (e.g., $S = \int_0^T D(t)dt$), A is baseline productivity, and u collects unobserved shocks and approximation error.

Interpretation. Equation (4) is not a claim that plant physiology is Cobb–Douglas; it is a tractable reduced-form that (a) nests the null $\theta = 0$, (b) supports clear welfare and adoption comparisons, and (c) is consistent with a first-order log approximation to a broad class of smooth technologies.

Elasticity and Marginal Product of Acoustic Input

Taking the logs of (4) gives

$$\ln Q = \ln A + \alpha \ln L + \beta \ln K + \gamma \ln W + \delta \ln N + \theta \ln S + u \quad (5)$$

The parameter of interest is the output elasticity w.r.t. the acoustic dose:

$$\theta = \frac{\partial \ln Q}{\partial \ln S} \quad (6)$$

For a given operating point (Q, S), the marginal product is

$$\frac{\partial Q}{\partial S} = \theta \frac{Q}{S} \quad (7)$$

This links agronomic effect sizes directly to economics: even small θ can be meaningful when (i) S can be delivered cheaply at scale (low unit cost of treatment) and (ii) the intervention does not increase other variable costs.

Costs, Net Returns, and an Adoption Threshold

Let p be the output price, and let $C_S(S)$ denote the cost of delivering dose S (equipment, energy, maintenance, labor). Conditional on other inputs, the profit contribution of acoustic stimulation is

$$\Delta\pi(S) = p(Q(S) - Q(0)) - C_S(S). \quad (8)$$

In a deterministic static setting, a sufficient condition for adoption at the margin is

$$p \cdot \left.\frac{\partial Q}{\partial S}\right|_{S=S^{\star}} \geq C_S'(S^{\star}) \quad (9)$$

This makes clear what the empirical design must identify: the causal effect of S on Q (hence θ) and the relevant cost schedule $C_S(\cdot)$.

Stochastic Environments

Because $\varepsilon(t)$ reflects weather and biological shocks, the relevant object for farmers is expected utility or risk-adjusted returns. In later sections, one can extend (4) to stochastic climate states and derive adoption under risk aversion, allowing acoustic stimulation to affect not only mean output ($E[Q]$) but also potentially yield variance (e.g., via stress-response pathways suggested in the mechanobiology literature) [14,15].

A Stochastic Production Environment

To connect agronomic variability to economic adoption, one can embed the production function in a stochastic environment where shocks affect realized output and where acoustic stimulation may shift both mean productivity and yield risk.

Lognormal Production with Mean and Risk Channels

Let the output be

$$Q = A\ L^{\alpha} K^{\beta} W^{\gamma} N^{\delta} S^{\theta} \exp(\varepsilon) \tag{10}$$

where ε captures within-season environmental realizations (weather, pest pressure, microclimate) and measurement noise. Assume

$$\varepsilon \sim \mathcal{N}\left(-\frac{1}{2}\sigma^2, \sigma^2\right) \tag{11}$$

so that $E[\exp(\varepsilon)] = 1$ and A retain the interpretation of baseline mean productivity. Define the deterministic component

$$\mu(S) \equiv AL^{\alpha} K^{\beta} W^{\gamma} N^{\delta} S^{\theta}.$$

Then,

$$E[Q \mid S] = \mu(S) \tag{12}$$

$$\mathrm{Var}(Q \mid S) = \mu(S)^2(\exp(\sigma^2) - 1) \tag{13}$$

Allowing Acoustic Stimulation to Affect Yield Risk

To allow for a stabilization channel, let the shock variance depend on the acoustic dose:

$$\sigma^2(S) = \sigma_0^2 - \kappa \ln S, \kappa \geq 0 \tag{14}$$

Where $S > 0$ and $\sigma_0^2 - \kappa \ln S \geq 0$ for all S.

interpreting $\kappa > 0$ as risk-reducing effects (e.g., via stress-response pathways) over the domain studied. More flexibly, one can treat $\sigma^2(S)$ as an estimable function (e.g., piecewise constant by treatment arm in an RCT). The key implication is that acoustic stimulation can increase welfare through (i) higher mean output (via θ) and/or (ii) reduced output variance (via $\sigma^2(S)$).

Farmer Decision-Making Under Uncertainty, Learning, and Diffusion

This section derives adoption incentives for acoustic stimulation when output is stochastic, farmers are risk averse, adoption may require an (at least partly) irreversible investment, and beliefs about treatment efficacy evolve with evidence. The framework is intentionally modular: it yields (i) a risk-adjusted static adoption rule, (ii) a two-period real-options formulation with experimentation, and (iii) a diffusion mechanism under heterogeneous priors and signals.

Stochastic Profits and Risk Preferences

Let the acoustic dose be $S \geq 0$ and let the output be random conditional on S:

$$Q(S) = \mu_Q(S)\exp(\varepsilon(S)), \varepsilon(S) \sim \mathcal{N}\left(-\frac{1}{2}\sigma^2(S), \sigma^2(S)\right) \tag{15}$$

so that $E[\exp(\varepsilon(S))] = 1$ and therefore

$$E[Q(S)] = \mu_Q(S), \mathrm{Var}(Q(S)) = \mu_Q(S)^2(\exp(\sigma^2(S)) - 1) \tag{16}$$

The function $\mu_Q(S)$ captures mean productivity effects; $\sigma^2(S)$ allows acoustic stimulation to alter yield volatility (a stabilization channel).

Let profit be

$$\Pi(S) = pQ(S) - C(S) - C_0 \tag{17}$$

where p is the output price, $C(S)$ is the cost of delivering dose S (possibly nonlinear), and C_0 is independent of S.

The farmer maximizes expected utility:

$$\max_{S\geq 0} \mathbb{E}[\mathrm{U}(\Pi(S))] \tag{18}$$

Assume CRRA preferences over (positive) profit:

$$\mathrm{U}(\pi) = \begin{cases} \dfrac{\pi^{1-\rho}}{1-\rho}, & \rho \neq 1, \\ \ln(\pi), & \rho = 1, \end{cases} \tag{19}$$

FOCs

For an interior optimum, differentiating under the expectation yields:

$$\frac{\mathrm{d}}{\mathrm{dS}}\mathbb{E}[\mathrm{U}(\Pi(S))] = \mathbb{E}\left[\mathrm{U}'(\Pi(S))\left(\mathrm{p}\frac{\partial \mathrm{Q}(S)}{\partial S} - \mathrm{C}'(\mathrm{S})\right)\right] = 0. \tag{20}$$

This condition is state-weighted: marginal benefits are valued more in low-profit states where $\mathrm{U}'(\Pi)$ is high.

Certainty-equivalent

For transparent adoption thresholds, use a second-order approximation of expected utility around $\mu_\Pi(S) = E[\Pi(S)]$:

$$\mathrm{E}[\mathrm{U}(\Pi)] \approx \mathrm{U}(\mu_\Pi) + \frac{1}{2}\mathrm{U}''(\mu_\Pi)\sigma_\Pi^2, \sigma_\Pi^2(\mathrm{~S}) = \mathrm{Var}(\Pi(S)) \tag{21}$$

Define absolute risk aversion $r_A(x) = -U''(x)/U'(x)$. The associated certainty equivalent can be written as:

$$\mathrm{CE}_\Pi(S) \approx \mu_\Pi(S) - \frac{1}{2}\mathrm{r}_A(\mu_\Pi(S))\sigma_\Pi^2(S) \tag{22}$$

Under CRRA, $\mathrm{r_A}(\mathrm{x}) = \rho/\mathrm{x}$, so

$$\mathrm{CE}_\Pi(S) \approx \mu_\Pi(S) - \frac{\rho}{2}\frac{\sigma_\Pi^2(S)}{\mu_\Pi(S)} \tag{23}$$

Since $\Pi(S) = pQ(S) - C(S) - C_0$ and $C(S)$ is deterministic conditional on S,

$$\mu_\Pi(S) = p\mu_Q(S) - C(S) - C_0 \tag{24}$$

$$\sigma_\Pi^2(S) = p^2\mathrm{Var}(Q(S)) = p^2\mu_Q(S)^2(\exp(\sigma^2(S)) - 1) \tag{25}$$

Substituting gives an operational risk-adjusted objective:

$$\mathrm{CE}_\Pi(S) \approx p\mu_Q(S) - C(S) - C_0 - \frac{\rho}{2}\frac{p^2\mu_Q(S)^2(\exp(\sigma^2(S)) - 1)}{p\mu_Q(S) - C(S) - C_0} \tag{26}$$

Risk-adjusted marginal adoption condition

A sufficient local condition for increasing S is $\mathrm{dCE}_{\Pi}(S)/dS \geq 0$. Differentiating (26) yields:

$$p\mu_Q'(S) - C'(S) - \frac{\frac{\rho}{2}\frac{p^2}{\mu_\Pi(S)^2}\left(\mu_\Pi(S)\frac{d}{dS}\mathrm{Var}\big(Q(S)\big) - \mathrm{Var}\big(Q(S)\big)\mu_\Pi'(S)\right)}{\mu_\Pi(S)^2} \geq 0 \tag{27}$$

where $\mu_\Pi'(S) = p\mu_Q'(S) - C'(S)$ and $\mathrm{Var}(Q(S))$ is given by (16). If $\sigma^2(S)$ falls with S (stabilization), then $d\mathrm{Var}(Q(S))/dS$ may be negative, raising the net marginal benefit of adoption under risk aversion.

Elasticity Representation and a Break-Even Point (BEP)

Suppose mean output is locally Cobb–Douglas in S holding other inputs fixed:

$$\mu_Q(S) = \bar{Q}S^\theta, \theta \geq 0 \tag{28}$$

implying

$$\mu_Q'(S) = \theta\frac{\mu_Q(S)}{S} \tag{29}$$

Risk-Neutral Benchmark

Under risk neutrality ($\rho = 0$), the marginal condition reduces to

$$p\theta\frac{\mu_Q(S)}{S} \geq C'(S) \tag{30}$$

or equivalently, the elasticity threshold

$$\theta \geq \frac{C'(S)S}{p\mu_Q(S)} \tag{31}$$

Risk-Adjusted Break-Even Point

For a discrete choice between $S = 0$ and $S = \bar{S}$, adoption occurs if

$$\mathrm{CE}_\Pi(\bar{S}) \geq \mathrm{CE}_\Pi(0) \tag{32}$$

where $\mathrm{CE}_\Pi(\cdot)$ is given in (23) or (26). This decomposes adoption into a mean gain offset by a risk penalty change:

$$\big(\mu_\Pi(\bar{S}) - \mu_\Pi(0)\big) \geq \frac{\rho}{2}\left(\frac{\sigma_\Pi^2(\bar{S})}{\mu_\Pi(\bar{S})} - \frac{\sigma_\Pi^2(0)}{\mu_\Pi(0)}\right) \tag{33}$$

Variance reduction (lower $\sigma^2(\bar{S})$) makes the right-hand side smaller, relaxing the mean-effect requirement for adoption.

Irreversible Adoption with Learning

Many technologies require upfront capital (devices, installation) that is partially irreversible. Let $F > 0$ denote a fixed adoption cost paid once upon adoption. After paying F, the farmer can choose a dose S each season at variable cost $C(S)$; to keep the notation simple, interpret $C(S)$ below as the variable component only.

Let the farmer be uncertain about the mean elasticity θ. A two-period structure captures the option value of waiting and/or experimenting:

- Period 0: the farmer chooses whether to (i) adopt now, (ii) run a pilot/experiment (at cost c_{test}) and decide the next period, or (iii) wait without experimenting.

- Period 1: the farmer chooses whether to adopt given updated beliefs.

Payoffs Conditional on Beliefs

Let $\mathrm{V}(\theta)$ denote the (risk-adjusted) per-period net surplus from adoption excluding the fixed cost F, evaluated at the optimal dose choice:

$$V(\theta) \equiv \max_{S\geq 0} \mathrm{CE}_{\Pi}(S;\theta) \tag{34}$$

where $\mathrm{CE}_{\Pi}(S;\theta)$ uses (26) with $\mu_{\mathrm{Q}}(\mathrm{S})$ parameterized by θ (e.g., (28)). Let $\beta \in (0,1)$ be the discount factor. If adoption is irreversible, the value of adopting the immediately given belief state b (a distribution over θ) is

$$\mathcal{A}(b) = \mathbb{E}_b[V(\theta)] - F + \beta\mathbb{E}_b[V(\theta)] \tag{35}$$

If the farmer waits one period without new information, the value is

$$\mathcal{W}(b) = \beta \max\{0, \mathbb{E}_b[V(\theta)] - F\} \tag{36}$$

where the max reflects the option to never adopt.

Experimentation and Posterior Updating

Assume a normal prior:

$$\theta \sim \mathcal{N}(\mu_0, \tau_0^2) \tag{37}$$

The farmer can observe a signal from pilot evidence (own plots, extension trial results, neighbor outcomes):

$$\grave{\theta} \,\Big|\, \theta \sim \mathcal{N}\left(\theta, \frac{\sigma_\theta^2}{n}\right) \tag{38}$$

where n is the effective sample size and σ_θ^2 is noise.
Posterior updating yields:

$$\tau_1^2 = \left(\frac{1}{\tau_0^2} + \frac{n}{\sigma_\theta^2}\right)^{-1}, \tag{39}$$

$$\mu_1 = \tau_1^2\left(\frac{\mu_0}{\tau_0^2} + \frac{n}{\sigma_\theta^2}\grave{\theta}\right). \tag{40}$$

Let $b_1(\cdot)$ denote the posterior belief distribution induced by observing $\grave{\theta}$.

Option Value of Information and the Experimentation Threshold

If the farmer experiments in period 0 at cost c_{test}, then in period 1, she adopts if $\mathbb{E}_{b_1}[\mathrm{V}(\theta)] \geq \mathrm{F}$. The value of experimentation can be written as:

$$\mathcal{E}(\mathrm{b}_0) = -c_{\text{test}} + \beta\mathbb{E}_{\grave{\theta}\sim \mathrm{pred}(\mathrm{b}_0)}\left[\max\left\{0, \mathbb{E}_{\mathrm{b}_1}[\mathrm{V}(\theta)] - \mathrm{F}\right\}\right], \tag{41}$$

where $\mathrm{pred}(b_0)$ is the predictive distribution of $\grave{\theta}$ under the prior b_0.
The value of information (VOI) from experimentation relative to waiting can be written as:

$$\mathrm{VOI}(\mathrm{b}_0) \equiv \mathcal{E}(\mathrm{b}_0) - \mathcal{W}(\mathrm{b}_0) \tag{42}$$

A farmer experiments when $\mathrm{VOI}(\mathrm{b}_0) \geq 0$. Higher precision (larger n or smaller σ_θ^2) increases VOI by reducing posterior uncertainty (39), thereby lowering the probability of mistaken adoption/rejection.

Adoption Rule with Irreversibility

Combining the above, the farmer chooses among adopt now, experiment, or wait:

$$\max\{\mathcal{A}(\mathrm{b}_0), \mathcal{E}(\mathrm{b}_0), \mathcal{W}(\mathrm{b}_0)\} \tag{43}$$

Irreversibility ($F > 0$) creates an option value of waiting even when $\mathbb{E}_{b_0}[V(\theta)]$ is slightly above F because postponing can avoid paying F when future information reveals low true effectiveness.

Diffusion with Heterogeneous Priors and Social Learning

To map micro adoption rules into diffusion dynamics, consider a population of farmers indexed by i with heterogeneous priors:

$$\theta \sim \mathcal{N}(\mu_{i0}, \tau_{i0}^2), (\mu_{i0}, \tau_{i0}^2) \sim H \tag{44}$$

where H is the cross-sectional distribution of prior means and variances (capturing differences in soils, crops, attitudes, and prior exposure to agronomic claims).

Static Diffusion Curve (Belief Heterogeneity Only)

Suppose each farmer faces a (risk-adjusted) threshold $\theta_i^\star$ (from (31) or (33)). If adoption is decided based on posterior mean μ_{i1}, then the adoption indicator after observing evidence is

$$\mathbf{1}\{\mu_{i1} \geq \theta_i^\star\}$$

Hence, the adoption rate in the population is

$$\Lambda_1 = \Pr(\mu_{i1} \geq \theta_i^\star) \tag{45}$$

which is increasing in the signal $\hat{\theta}$ and in experimental precision (through τ_{i1}^2) and decreasing in costs (through $\theta_i^\star$).

Dynamic Diffusion with Accumulating Evidence

Let t index seasons. Each season, a subset of farmers observes additional signals (own pilots, neighbors, extension trials):

$$\hat{\theta}_{it} = \theta + \eta_{it}, \eta_{it} \sim \mathcal{N}\left(0, \frac{\sigma_\theta^2}{n_{it}}\right) \tag{46}$$

Updating sequentially, posterior precision increases over time:

$$\frac{1}{\tau_{i,t}^2} = \frac{1}{\tau_{i,t-1}^2} + \frac{n_{it}}{\sigma_\theta^2} \tag{47}$$

and posterior means evolve as a weighted average of past means and new signals. Define adoption at time t by the threshold rule $\mu_{i,t} \geq \theta_i^\star$ (or by the real-options policy (43) when irreversibility matters). Then, the diffusion path is

$$\Lambda_t = \Pr\left(\text{ adopt at } t \mid H, \{\hat{\theta}_{is}\}_{s \leq t}\right) \tag{48}$$

which typically rises as evidence accumulates and posterior variances shrink.

Networked Social Learning (Optional Extension)

Let each farmer observe m_{it} neighbor outcomes, which provide signals with an effective sample size proportional to the number of treated observations in the local network. A reduced-form representation is

$$n_{it} = n_{it}^{\text{own}} + \lambda m_{it}, \lambda \in [0,1], \tag{49}$$

where λ captures the informativeness of neighbor data (attenuated by differences in conditions). This creates endogenous diffusion: as more neighbors adopt (higher m_{it}), information improves and adoption accelerates.

Empirical Mapping

The chapter identifies a minimal set of empirical objects required for disciplined policy evaluation and scaling:

- Mean effect: $\mu_Q(S)$ (or elasticity θ) as a causal function of dose.
- Risk effect: $\sigma^2(S)$ (or $d\mathrm{Var}(Q(S))/dS$) to quantify stabilization.
- Cost schedule: $C(S)$ and $C'(S)$ (including fixed cost F if adoption is capital-intensive).
- Preference/price shifters: ρ and p (or proxies), to translate agronomic effects into welfare and adoption.
- Beliefs and learning: $(\mu_{\mathrm{i0}}, \tau_{\mathrm{i0}}^2)$ and signal precision $(\sigma_\theta^2, \mathrm{n_{it}})$ to predict diffusion.

These components jointly discipline the RCT design and structural calibration: the experiment must measure not only average treatment effects on yields but also changes in yield dispersion and the engineering cost of dose delivery while collecting enough information to parameterize adoption behavior and learning dynamics.

Causal Identification Strategy

Potential Outcomes, Estimands, and Assumptions

One can adopt the Neyman-Rubin potential outcomes framework [19,20]. Let experimental units be trays (clusters) indexed by $\mathrm{j} = 1, \ldots$, J, each containing plants $\mathrm{i} = 1, \ldots, m_j$. Treatment is assigned at the tray level. Let $Z_j \in \mathcal{Z}$ denote the assigned acoustic condition (e.g., silence, tone, harmonic spectrum, white noise), and define $z = 0$ as silence.

SUTVA at the tray level. Let $Y_{ij}(z)$ be the potential outcome (e.g., final dry biomass or yield) for plant i in tray j if tray j is assigned treatment z. One can assume (i) no hidden versions of treatment (the protocol delivers a well-defined acoustic dose for each arm) and (ii) no interference across trays (acoustic spillovers are prevented by physical isolation, distance, and shielding). Under this assumption, $\mathrm{Y_{ij}(z)}$ depends only on z assigned to tray j.

The observed outcomes satisfy:

$$Y_{ij} = Y_{ij}(Z_j) \tag{50}$$

Primary estimand (tray-level ATE). Because assignment is at the tray level, the cleanest estimand is defined on tray averages:

$$\overleftarrow{Y}_j(z) \equiv \frac{1}{m_j}\sum_{i=1}^{m_j} Y_{ij}(z), \tau(z) \equiv \mathbb{E}\left[\overleftarrow{Y}_j(z) - \overleftarrow{Y}_j(0)\right] \tag{51}$$

for each active $\mathrm{arm} z \in \mathcal{Z} \setminus \{0\}$. If m_j varies, this estimand weights trays equally; alternative weighting (e.g., by $\mathrm{m_j}$) can be prespecified.

Identification under random assignment. Let the assignment mechanism be completely randomized or blocked randomized at the tray level, independent of potential outcomes:

$$Z \perp \left\{\overleftarrow{Y}_j(z) : z \in \mathcal{Z}, j = 1, \ldots, J\right\} \tag{52}$$

Then, difference-in-means estimators for each contrast are unbiased for $\tau(z)$.

Multiarm RCT: Estimators and Multiple Testing

Let J_z denote the number of trays assigned to arm z. For each $z \neq 0$, define the (tray-level) difference-in-means estimator:

$$\hat{\tau}(z) = \left(\frac{1}{J_z}\sum_{j:Z_j=z} \overleftarrow{Y_j}\right) - \left(\frac{1}{J_0}\sum_{j:Z_j=0} \overleftarrow{Y_j}\right) \tag{53}$$

where $\overleftarrow{Y_j} = \frac{1}{m_j}\sum_i Y_{ij}$ is the observed tray mean.

Because there are multiple arms, one can prespecify a family of hypotheses:

$$\mathrm{H}_{0,z}: \tau(z) = 0 \text{ for each } z \neq 0$$

and control familywise error (FWER) across these contrasts using either (i) randomization-based stepdown procedures or (ii) a conservative Holm adjustment on p values.

Covariate Adjustment and Blocking

Let X_j denote pretreatment tray covariates (seed batch, planting date/time, initial weight, position in growth chamber). Randomization guarantees unbiasedness without covariates, but covariate adjustment can improve precision. A standard ANCOVA specification at the tray level can be written as:

$$\overleftarrow{Y_j} = \alpha + \sum_{z\neq 0} \beta_z \mathbf{1}\{Z_j = z\} + \gamma' X_j + \lambda_b + u_j \tag{54}$$

where λ_b are block fixed effects if assignment is blocked by the seed batch (or chamber shelf). Inference is still design-based; β_z estimates $\tau(z)$ when the model is saturated in treatment indicators and blocks.

Clustered Design and Inference

Why Tray-Level Analysis is Primary

Because treatment is assigned at the tray level, the effective sample size is the number of trays, not the number of plants. Plant-level regressions must account for clustering; equivalently, one can analyse tray means directly.

Tray-level regression Using tray means, the regression in (54) produces the same point estimates as plant-level OLS with tray-clustered standard errors when trays are equally weighted
and m_j is constant.

Cluster-Robust Standard Errors and Small-J Corrections

If one can estimate plant-level models,

$$Y_{ij} = \alpha + \sum_{z\neq 0} \beta_z 1\{Z_j = z\} + \gamma' X_{ij} + \lambda_b + \varepsilon_{ij} \tag{55}$$

One can compute cluster-robust variance at the tray level:

$$\widehat{\mathrm{Var}}(\hat{\beta}) = (X'X)^{-1}\left(\sum_{j=1}^{J} X_j' \grave{u}_j \grave{u}_j' X_j\right)(X'X)^{-1} \tag{56}$$

where X_j stacks regressors for all plants in tray j and $\grave{u}_j$ stacks residuals. Because greenhouse experiments often have modest numbers of trays, one can apply finite-sample corrections and few-cluster robust inference: (i) t tests using J-df degrees of freedom, and (ii) a wild cluster bootstrap as a robustness check for p values and confidence intervals.

Randomization Inference

For each contrast z vs. 0, one can report randomization-inference p-values under the sharp null:

$$\mathrm{H}_{0,z}^{\text{sharp}} : \overleftarrow{Y}_{\mathrm{j}}(z) = \overleftarrow{Y}_{\mathrm{j}}(0) \forall \mathrm{j}$$

Let $\hat{\tau}_{\text{obs}}(z)$ be the observed statistic from (53). Over the set of admissible assignments $\mathcal{A}$ induced by the randomization protocol (including blocking), the two-sided p value can be written as:

$$\mathrm{p}_z = \frac{1}{|\mathcal{A}|} \sum_{\mathrm{a} \in \mathcal{A}} 1(|\hat{\tau}_{\mathrm{a}}(z)| \geq |\hat{\tau}_{\mathrm{obs}}(z)|) \tag{57}$$

One can also compute step-down randomization inference to control FWER across multiple arms.

Experimental Design Details

Units, Arms, and Implementation

Crop and outcome. Crop: Vigna radiata. Primary outcome: harvestable yield proxy (preregistered), such as dry biomass or seed mass at T.

Treatment arms.

Let $\mathcal{Z} = \{0,1,2,3\}$:

$z = 0$: Silence (control)

$z = 1$: $220\ \mathrm{Hz}$ pure tone

$z = 2$: Harmonic spectrum $(200 - 2000\ \mathrm{Hz})$

$z = 3$: White noise

Acoustic dose monitoring. The sound intensity at canopy height is targeted at $85 \pm 5\ \mathrm{dB}$ (SPL) with continuous logging. Dose compliance is checked by (i) sensor calibration pretrial, (ii) periodic validation measurements, and (iii) automated alerts for deviations outside tolerance.

Blocking, Balance, and Protocol Controls

Blocking. Randomization is blocked by the seed batch (and optionally by the chamber position) to reduce variance:

Within each block, $\mathrm{b}, \#\{\mathrm{Z}_{\mathrm{j}} = z\}$ is fixed for all $z \in \mathcal{Z}$.

Environmental controls. Light, humidity, irrigation, and temperature are held constant across trays using standardized schedules and monitored sensors. Any time-varying chamber shocks (e.g., temperature drift) are captured by block/time indicators and logged to support diagnostics.

Preanalysis plan. One can prespecify: primary outcome, primary contrasts (each z vs. 0), variance estimators (cluster-robust + RI), and a multiplicity correction approach.

Power Under Cluster Randomization

Model and Intracluster Correlation

Assume the standard random-effects decomposition:

$$Y_{ij} = \mu + \sum_{z \neq 0} \beta_z 1\{Z_j = z\} + u_j + \epsilon_{ij} \tag{58}$$

with $\mathrm{u_j} \sim (0, \sigma_{\mathrm{u}}^2)$ and $\epsilon_{\mathrm{ij}} \sim (0, \sigma_\epsilon^2)$ independent. The intracluster correlation (ICC) is

$$\rho = \frac{\sigma_{\mathrm{u}}^2}{\sigma_{\mathrm{u}}^2 + \sigma_\epsilon^2} \tag{59}$$

Design Effect and Effective Sample Size

If $m_j = m$ for all trays, the design effect is

$$\mathrm{DE} = 1 + (m - 1)\rho \tag{60}$$

and an effective plant-level sample size representation is

$$\mathrm{n_{eff}} = \frac{\mathrm{Jm}}{1 + (\mathrm{m} - 1)\rho} \tag{61}$$

However, inference and power are ultimately driven by J (the number of trays), so power calculations are conducted at the tray level whenever possible.

MDE for a Two-Arm Contrast (At the Tray Level)

For a single contrast between z and 0 using tray means and equal allocation $\mathrm{J}_z = \mathrm{J}_0 = \mathrm{J}/2$, the minimum detectable effect (MDE) in outcome units is approximately:

$$\mathrm{MDE} = \left(z_{1-\alpha/2} + z_{1-\beta}\right)\sqrt{\frac{2\sigma_{\bar{\mathrm{Y}}}^2}{J/2}}, \sigma_{\bar{\mathrm{Y}}}^2 = \frac{\sigma_\epsilon^2}{m} + \sigma_u^2 \tag{62}$$

where $\sigma_{\bar{\mathrm{Y}}}^2$ is the variance of tray means.
For multiple treatment arms, one can compute MDE for each z vs. 0 using its actual allocation J_z and adjust α to reflect multiplicity (e.g., Holm-adjusted critical values or an RI-based stepdown).

Elasticity Mapping from Experimental Effects

Let the experimental estimand be a proportional mean effect for arm z:

$$\delta_z \equiv \frac{\mathbb{E}[Q \mid Z = z] - \mathbb{E}[Q \mid Z = 0]}{\mathbb{E}[Q \mid Z = 0]} \tag{63}$$

In practice, δ_z can be estimated via a $\log$ specification:

$$\ln\ Q_{ij} = \alpha + \sum_{z \neq 0} \beta_z \mathbf{1}\{Z_j = z\} + \lambda_b + \varepsilon_{ij} \tag{64}$$

where β_z is an approximate log point effect (clustered inference), so that $\hat{\delta}_z \approx \exp\left(\hat{\beta}_z\right) - 1$. To map this into a production elasticity w.r.t. an acoustic dose measure S, define the dose ratio between $\mathrm{arm} z$ and the control:

$$\kappa_z \equiv \frac{S_z}{S_0}$$

Under a local Cobb–Douglas relationship in S for mean output, $E[Q] \propto S^\theta$, the implied elasticity satisfies:

$$\theta \approx \frac{\ln(1+\delta_z)}{\ln(\kappa_z)} \tag{65}$$

Note*: The elasticity mapping is mathematically invalid if the control arm is silence, since $\kappa_z = S_z / S_0$ is undefined.

Delta-method variance propagation yields:

$$\widehat{\mathrm{Var}}(\hat{\theta}) \approx \left(\frac{1}{\ln(\kappa_z)} \cdot \frac{1}{1+\delta_z}\right)^2 \widehat{\mathrm{Var}}(\hat{\delta}_z) \tag{66}$$

Note*: Inherits the same invalidity as Eq. (65): $\ln(\kappa_z)$ is undefined when S_0=0.

with confidence intervals computed either by (i) the delta method, (ii) bootstrap at the tray level, or (iii) randomization inversion.

Structural Adoption Simulation

Adoption Rule

Let farmers adopt if the (risk-adjusted) net benefit is nonnegative. A parsimonious risk-neutral threshold in elasticity space can be written as:

$$\theta \geq \theta^\star(r) \equiv \frac{C'(S_r)S_r}{p_r \mu_{Q,r}(S_r)} \tag{67}$$

where r indexes the region/crop system, prices p_r, baseline mean output $\mu_{Q,r}$, and the relevant dose point S_r at which the marginal cost is evaluated (consistent with engineering delivery technology). Under risk aversion, $\theta^\star(r)$ is adjusted using the certainty-equivalent condition derived earlier, allowing variance effects $\sigma^2(S)$ to shift the threshold.

Uncertainty, Heterogeneity, and Monte Carlo

Let θ be estimated with uncertainty from the RCT: $\theta \sim \mathcal{N}(\hat{\theta}, \hat{V}_\theta)$ (or the randomization-based confidence set). Let the electricity price and crop price be random:

$$p_{e,r} \sim F_{e,r}, p_r \sim F_{p,r}$$

Let the variable cost be a function $\mathrm{C}(\mathrm{S}; \mathrm{p}_{e,r})$ derived from the device power demand, runtime, and amortization.

Define adoption probability in region r as:

$$\Pr(\text{ adopt } \mid r) = \iiint \mathbf{1}\left(\theta \geq \theta^\star(r; p_{e,r}, p_r)\right) dF_\theta dF_{e,r} dF_{p,r} \tag{68}$$

One can compute (68) by Monte Carlo simulation and report sensitivity to (i) device efficiency, (ii) baseline yields, and (iii) the variance-reduction channel.

Welfare Decomposition

Let Y denote the relevant payoff object (profit or consumption). Consider the welfare gain from moving from $S = 0$ to $S = \bar{S}$:

$$\Delta \mathrm{EU} \equiv \mathbb{E}[\mathrm{U}(\mathrm{Y}_{\bar{S}})] - \mathbb{E}[\mathrm{U}(\mathrm{Y}_0)].$$

Using a second-order approximation around $\mu_Y = \mathrm{E}[\mathrm{Y}]$:

$$\Delta \mathrm{EU} \approx \mathrm{U}'(\mu_Y)\Delta \mathrm{E}[\mathrm{Y}] + \frac{1}{2}\mathrm{U}''(\mu_Y)\Delta \mathrm{Var}(\mathrm{Y}). \tag{69}$$

Under CRRA, $\mathrm{U}'(\mathrm{y}) = \mathrm{y}^{-\rho}$ and $\mathrm{U}''(\mathrm{y}) = -\rho \mathrm{y}^{-\rho-1}$, so

$$\Delta \mathrm{EU} \approx \mu_Y^{-\rho} \Delta \mathrm{E}[\mathrm{Y}] - \frac{\rho}{2}\mu_Y^{-\rho-1} \Delta \mathrm{Var}(\mathrm{Y}) \tag{70}$$

Thus, holding mean effects fixed, a reduction in payoff variance $(\Delta\mathrm{Var}(Y) < 0)$ increases expected utility.

Market-Level Considerations

If adoption becomes widespread, aggregate supply shifts. Let aggregate output be

$$Q^S = \int Q_i di.$$

Let inverse demand be $p = P(Q^S)$ with price elasticity of demand $\varepsilon_d \equiv -\frac{dQ}{dp}\frac{p}{Q}$. A small proportional supply shift dQ/Q induces a proportional price change:

$$\frac{dp}{p} \approx -\frac{1}{\varepsilon_d}\frac{dQ}{Q}. \tag{71}$$

One can incorporate (71) into adoption and welfare simulations by updating p_r endogenously when predicted adoption implies nontrivial supply expansion. This distinguishes (i) producer surplus effects (possibly dampened by falling prices), (ii) consumer surplus effects (potentially increased by lower prices), and (iii) distributional impacts across adopters and nonadopters.

Implementation Timeline and Milestones (18 Months)

This project is organized as a sequence of gated phases. Progression to each subsequent phase is conditional on meeting prespecified technical and data-quality criteria.

Phase 1: Infrastructure, Compliance, and Calibration (Months 1-3)

- Procurement and verification: Acquire acoustic emission systems, SPL meters, and data loggers; verify factory calibration with reference signals.
- Physical isolation: Construct sound-isolated greenhouse compartments (or isolation enclosures) with documented attenuation curves across relevant frequencies.
- Dose definition and monitoring: Prespecify the delivered acoustic dose metric S (e.g., time-integrated intensity at canopy height, optionally frequency-weighted) and implement continuous logging for compliance.
- Environmental baselining: Benchmark temperature, humidity, CO_2, light, and irrigation uniformity; document tolerances and sensor drift.
- Input standardization: Baseline soil media analysis and seed viability testing by batch; preregister handling and planting protocol.

Gate/Milestone 1 (Go/No-Go): (i) acoustic stability within ± 3 dB at canopy height for each arm over a 48-hour stress test; (ii) cross-compartment acoustic leakage below a prespecified threshold (e.g., <X dB above ambient in control compartments at treatment frequencies); (iii) environmental control within tolerance bands (temperature, humidity, light) with complete sensor logs.

Phase 2: Pilot Wave (Months 4-6)

- Pilot RCT: Conduct a small-scale blocked randomized pilot (e.g., $\mathrm{J} = 20$ trays) to test feasibility, compliance, and measurement reproducibility.
- Measurement reliability: Validate primary outcomes (e.g., dry biomass/seed mass) with duplicate measurement for a subset; estimate measurement error variance.
- Design parameter estimation: Estimate ICC $\hat{\rho}$, tray-mean variance $\widehat{\sigma^2_{\bar{Y}}}$, attrition rates, and compliance deviations to recalibrate power.

- Data pipeline dry run: End-to-end ETL, version control, audit logs, and reproducible report generation.

Gate/Milestone 2: (i) preanalysis plan (PAP) finalized and time-stamped; (ii) updated power analysis using pilot ICC and variance estimates; (iii) passing data-quality checks (missingness, protocol adherence, auditability).

Phase 3: Full RCT (Months 7-14)

- Three independent growth cycles: Implement three temporally separated cycles to test reproducibility across runs and mitigate cycle-specific shocks.
- Blocked randomization: Assign trays within blocks defined by seed batch and chamber position; store randomization seeds and assignment files.
- Continuous compliance monitoring: Real-time tracking of dose delivery and environment; automated flags for threshold breaches.
- Prespecified monitoring only: Interim analyses restricted to integrity metrics (missingness, compliance, sensor drift) unless the PAP explicitly allows adaptive reallocation.
- Gate/Milestone 3: (i) complete, locked dataset with documented provenance; (ii) replication package compiles end-to-end; (iii) prespecified primary analyses reproduced from raw logs without manual intervention.

Phase 4: Structural Calibration, External Validity, and Reporting (Months 15-18)

- Estimation: Estimate mean effects, variance effects, and implied elasticities (dose–response mapping).
- Economic calibration: Map agronomic estimates into adoption thresholds using observed energy/device cost schedules; run Monte Carlo simulations across price/energy scenarios.
- Welfare and policy outputs: Welfare decomposition under risk aversion; cost-effectiveness analysis for prioritized contexts (e.g., high-value greenhouse crops).
- Dissemination: Draft final report and policy brief; publish replication package and documentation.

Gate/Milestone 4: final paper + technical appendix; open replication archive; and a documented cost-effectiveness/adoption dashboard (parameterized, reproducible).

Governance, Transparency, and Reproducibility

The governance framework is designed to minimize researcher degrees of freedom and ensure the credibility of null results.

1. Preanalysis plan (PAP): Registered prior to Phase 3, specifying outcomes, contrasts, inference procedures (cluster-robust + randomization inference), multiplicity adjustments, exclusion criteria, and robustness checks.
2. Version-controlled research compendium: Code, data dictionaries, instrument calibration logs, and randomization seeds stored in a public repository upon publication (with embargo options if required by partners).
3. Reproducible pipelines: Automated scripts generate all tables/figures directly from raw sensor logs and measurement files; any manual edits are prohibited postlock.

4. Null-results commitment: Reporting follows prespecified hypotheses and includes null and adverse findings with equal prominence.
5. Replication incentives: Independent replication is encouraged via clear protocols, open materials lists, and unit-cost documentation for dose delivery.

Oversight. An oversight committee reviews gate criteria and PAP compliance:

- Agricultural economist (welfare/adoption calibration).
- Experimental statistician (design, power, inference).
- Plant physiologist (protocol plausibility, stress pathways).
- External ethics/sustainability advisor (energy use, reporting integrity).

Risk Assessment and Mitigation

Technical Risks

Acoustic spillovers/interference: Treatment leakage contaminates controls and attenuates estimates.

Mitigation: Physical isolation; frequency-specific attenuation tests; continuous SPL logging in control compartments; prespecified leakage thresholds triggering redesign; placebo checks using distance-to-source gradients.

Biological Risks

Null or highly heterogeneous responses: Effects may be zero on average or vary strongly by conditions.

Mitigation: Multicycle replication; prespecified heterogeneity analysis (e.g., by seed batch or baseline vigor); interpretation framed as disciplined falsification if null.

Economic Risks

Costs dominate benefits: Energy/device costs may exceed the value of yield gains in low-value crops.

Mitigation: Prioritize high-value controlled-environment crops first; estimate $C(S)$ precisely; identify contexts where energy is cheap/renewable and yield value is high.

Statistical Risks

Underpowered inference due to ICC: Effective sample size is the number of trays. Mitigation: Pilot-estimated ICC; tray-level primary analysis; power recalibration before Phase 3; conservative multiplicity adjustment; randomization inference for p values.

Projected Food Security and Poverty Relevance Aspects

To avoid overclaiming, one can present back-of-the-envelope calculations as scenario analysis that translates estimated elasticities into physical output under clearly stated assumptions.
Let baseline household production be Y_0 (tons/year). Suppose the intervention generates a proportional increase δ in output:

$$Y_1 = Y_0(1+\delta), \Delta Y = Y_0\delta \tag{72}$$

For illustration, if $\delta = 0.04$ and $Y_0 = 2$ tons/year, then $\Delta Y = 0.08$ tons/year. To translate into calories, let k denote kilocalories per ton for the crop/product mix (contextspecific). Then,

$$\Delta \text{ Calories } = \mathrm{k}\Delta \mathrm{Y}. \quad (73)$$

Aggregating across H households:

$$\Delta Y_{\text{agg}} = \mathrm{H}\Delta Y = \mathrm{H}Y_0\delta \quad (74)$$

These calculations are reported only as sensitivity bands because δ is to be estimated and because adoption will be incomplete and context-dependent (Section 14).

Scalability Pathway and External Validity

Scaling is treated as an economic and engineering question, not an assumption. One can structure scale-up as staged external-validity testing:

1. Stage 1: Controlled-environment validation (this project): identify causal mean and variance effects and map them to an elasticity θ and dose–response.
2. Stage 2: Commercial greenhouse partnership: test external validity under operational constraints (spacing, airflow, heterogeneous microclimates) and measure real-world compliance and costs.
3. Stage 3: Peri-urban horticulture pilots: evaluate performance where infrastructure is weaker and environmental volatility is higher.
4. Stage 4: Low-power decentralized units: feasibility study for solar-powered or off-grid acoustic delivery, conditional on Stage 2/3 results.

Energy feasibility criterion. Let the variable cost of dose be $C'(S)$ and the risk-neutral marginal revenue be $\mathrm{p}\partial\mu_{\mathrm{Q}}(\mathrm{S})/\partial S$. A necessary condition for scalable adoption at the margin can be written as:

$$C'(S) \leq p\frac{\partial\mu_Q(S)}{\partial S} = p\theta\frac{\mu_Q(S)}{S} \text{ (under Cobb-Douglas mean response).} \quad (75)$$

Under risk aversion, variance reduction can relax this criterion through the certainty-equivalent condition derived earlier.

Cost Budgeting

All costs are reported in USD and correspond to deliverables required for causal identification and reproducibility. Personnel costs are excluded under institutional co-funding.

Item	Cost (USD)
Carefully calibrated acoustic emission system(s) + SPL sensors/loggers	25,000
Environmental control (compartmentation, sensors, controllers)	30,000
Measurement instrumentation (scales, dryers, imaging if used)	20,000
Replication materials (seeds, media, trays, consumables)	10,000
Data infrastructure (storage, compute, backup, audit tooling)	5,000
Contingency (repairs, replacements, overruns)	10,000
Personnel Costs R&D	-
Personnel Costs Technicians, Organization, Support Roles	-

Total$\gg$ $100{,}000$
Cost transparency. For each line item, one can provide unit costs, vendor quotes, and depreciation assumptions used to construct $C(S)$ and $C'(S)$ for adoption simulations.

Monitoring and Performance Evaluation
Monitoring is divided into (i) integrity monitoring (to protect identification) and (ii) outcome monitoring (to estimate effects). Integrity monitoring does not use outcome data for decision making unless prespecified in the PAP.

Integrated Monitoring (Weekly)
- Dose compliance summaries (mean SPL and variance by tray/compartment; out-of-band incidents).
- Environmental stability (temperature/humidity/light/CO_2 drift; sensor failures).
- Data completeness (missingness, timestamp integrity, protocol deviations).
- Midpoint independent audit of raw logs, randomization files, and measurement chain of custody.

Outcome Metrics (Postharvest, Prespecified)
- Primary causal estimands: tray-level mean effects $\hat{\tau}(z)$ and dispersion effects (changes in variance/quantiles), with randomization inference.
- Economic viability: estimated $\hat{\theta}$ mapped to threshold conditions using observed $C(S)$ and scenario prices.
- Stabilization: estimated effects on $\sigma^2(S)$ or on distributional spread (e.g., interquartile range).
- Predicted adoption: Monte Carlo adoption probabilities by region and crop system.

Ethical and Sustainability Considerations
No human subjects are involved. The primary ethical and sustainability considerations relate to energy use, equipment safety, and transparent reporting.
- Energy accounting: kWh consumption is metered and reported; carbon intensity scenarios are used in scale-up evaluation.
- Safety and nuisance: acoustic exposure limits for staff are respected; compartments prevent harmful exposure and noise leakage.
- Open reporting: results are reported irrespective of sign; adverse outcomes (reduced growth) are documented.

Preliminary Conclusion
Global food insecurity persists under binding climatic and resource constraints, while traditional intensification margins are increasingly limited. Acoustic stimulation is an unconventional but empirically testable hypothesis. This proposal treats that hypothesis with unusually strong standards: a blocked multiarm RCT with cluster-robust and design-based inference, explicit dose compliance monitoring, and structural mapping from causal agronomic effects to adoption thresholds and welfare under risk.

The objective is disciplined evaluation rather than advocacy. If effects are null or economically dominated, the hypothesis is rejected with high credibility. If effects are modest but robust and cost-effective in well-defined contexts (likely controlled-environment, high-value production), the framework provides a defensible pathway for external validity testing and scale-up assessment.

Appendix: Derivations, Sensitivity, and Figures

A1. Risk-Neutral Adoption Threshold

Assume mean output (holding other inputs fixed) is locally Cobb–Douglas in acoustic dose:

$$\mu_{\mathrm{Q}}(S) = \overline{\mathrm{Q}}S^{\theta}, \theta \geq 0 \tag{76}$$

and let profit under risk neutrality be

$$\Pi(S) = p\mu_Q(S) - C(S) - C_0 \tag{77}$$

where p is the output price and $C(S)$ is the variable cost of delivering dose S. The first-order condition (interior optimum) is

$$\frac{\mathrm{d}\Pi(S)}{\mathrm{dS}} = \mathrm{p}\mu_{\mathrm{Q}}'(S) - \mathrm{C}'(\mathrm{S}) = 0 \tag{78}$$

From (76), $\mu_Q'(S) = \theta\mu_Q(S)/S$. Substituting into (78) yields

$$p\theta\frac{\mu_Q(S)}{S} = C'(S), \tag{79}$$

and therefore, the implied elasticity threshold at operating point S is

$$\theta_{\mathrm{RN}}^{\star}(S) \equiv \frac{\mathrm{C}'(S)S}{\mathrm{p}\mu_{\mathrm{Q}}(S)} \tag{80}$$

Note: if one assumes linear costs $\mathrm{C}(\mathrm{S}) = \mathrm{c}\mathrm{S}$, then $\mathrm{C}'(\mathrm{S}) = \mathrm{c}$ and (80) reduces to $\theta_{\mathrm{RN}}^{\star}(\mathrm{S}) = \mathrm{c}S/\big(p\mu_{\mathrm{Q}}(S)\big)$.

A2. Price Sensitivity of the Risk Neutral Threshold

Holding S and $\mu_Q(S)$ fixed, differentiate (80) w.r.t. p:

$$\frac{\partial\theta_{\mathrm{RN}}^{\star}}{\partial p} = -\frac{C'(S)S}{p^2\mu_Q(S)} < 0 \tag{81}$$

Higher crop prices lower the minimum elasticity required for adoption. If the marginal cost depends on electricity prices, $C'(S) = C'(S; p_e)$, then

$$\frac{\partial\theta_{RN}^{\star}}{\partial p_e} = \frac{S}{p\mu_Q(S)}\frac{\partial C'(S;p_e)}{\partial p_e} > 0 \text{ if } \partial C'/\partial p_e > 0 \tag{82}$$

A3. Certainty-Equivalent Adoption Condition under CRRA

Let realized profit be $\Pi(S) = pQ(S) - C(S) - C_0$, and define $\mu_\Pi(S) = E[\Pi(S)]$ and $\sigma_\Pi^2(S) = \mathrm{Var}(\Pi(S))$.

Using a second-order expansion of $\mathrm{E}[\mathrm{U}(\Pi)]$ around μ_Π,

$$\mathrm{E}[\mathrm{U}(\Pi)] \approx \mathrm{U}(\mu_\Pi) + \frac{1}{2}\mathrm{U}''(\mu_\Pi)\sigma_\Pi^2 \tag{83}$$

Define the certainty equivalent CE_Π as the scalar satisfying $\mathrm{U}(\mathrm{CE}_\Pi) \approx \mathrm{E}[\mathrm{U}(\Pi)]$. A standard approximation yields

$$\mathrm{CE}_\Pi(S) \approx \mu_\Pi(S) - \frac{1}{2}\mathrm{r}_A(\mu_\Pi(S))\sigma_\Pi^2(S) \tag{84}$$

where $r_A(x) = -U''(x)/U'(x)$ is absolute risk aversion. Under CRRA, $r_A(x) = \rho/x$, so

$$\mathrm{CE}_{\Pi}(\mathrm{S}) \approx \mu_{\Pi}(\mathrm{S}) - \frac{\rho}{2}\frac{\sigma_{\Pi}^{2}(\mathrm{~S})}{\mu_{\Pi}(\mathrm{S})} \tag{85}$$

For a discrete adoption choice between $S = 0$ and $S = \bar{S}$, adoption occurs if

$$\mathrm{CE}_{\Pi}(\bar{\mathrm{S}}) \geq \mathrm{CE}_{\Pi}(0) \tag{86}$$

i.e.,

$$\left(\mu_{\Pi}(\bar{S}) - \mu_{\Pi}(0)\right) \geq \frac{\rho}{2}\left(\frac{\sigma_{\Pi}^{2}(\bar{S})}{\mu_{\Pi}(\bar{S})} - \frac{\sigma_{\Pi}^{2}(0)}{\mu_{\Pi}(0)}\right) \tag{87}$$

Variance reduction reduces the risk penalty term on the right-hand side and therefore relaxes the mean-effect requirement for adoption.

A4. Lognormal Output: Mean and Variance

Assume lognormal shocks:

$$Q(S) = \mu_{Q}(S)\exp(\varepsilon(S)), \varepsilon(S) \sim \mathcal{N}\left(-\frac{1}{2}\sigma^{2}(S), \sigma^{2}(S)\right) \tag{88}$$

so that $E[\exp(\varepsilon(S))] = 1$ and hence

$$E[Q(S)] = \mu_{Q}(S) \tag{89}$$

The variance is

$$\mathrm{Var}(Q(S)) = \mu_{Q}(S)^{2}(\exp(\sigma^{2}(S)) - 1) \tag{90}$$

If acoustic treatment reduces the log-variance from σ_0^2 to $\sigma_1^2 = \sigma_0^2 - \Delta\sigma^2$ holding μ_{Q} fixed, then

$$\Delta\mathrm{Var}(\mathrm{Q}) = \mu_{\mathrm{Q}}^{2}[\exp(\sigma_{1}^{2}) - \exp(\sigma_{0}^{2})] = -\mu_{\mathrm{Q}}^{2}\exp(\sigma_{0}^{2})\left(1 - \exp(-\Delta\sigma^{2})\right) < 0 \tag{91}$$

A5. Mapping an Experimental Proportional Effect to an Elasticity

Let δ_z be the proportional mean effect of $\mathrm{arm} z$ relative to the control:

$$\delta_{z} \equiv \frac{\mathrm{E}[\mathrm{Q} \mid \mathrm{Z} = z] - \mathrm{E}[\mathrm{Q} \mid \mathrm{Z} = 0]}{\mathrm{E}[\mathrm{Q} \mid \mathrm{Z} = 0]} \tag{92}$$

Let the dose ratio between $\mathrm{arm} z$ and the control be $\kappa_z = S_z/S_0$. If the mean output is locally $\mathrm{E}[\mathrm{Q}] \propto S^{\theta}$, then

$$\frac{\mathrm{E}[\mathrm{Q} \mid \mathrm{Z} = z]}{\mathrm{E}[\mathrm{Q} \mid \mathrm{Z} = 0]} = \left(\frac{\mathrm{S}_{z}}{\mathrm{S}_{0}}\right)^{\theta} = \kappa_{z}^{\theta} \tag{93}$$

However, $\mathrm{E}[\mathrm{Q} \mid \mathrm{Z} = z]/\mathrm{E}[\mathrm{Q} \mid \mathrm{Z} = 0] = 1 + \delta_z$, hence

$$1 + \delta_{z} = \kappa_{z}^{\theta} \Rightarrow \theta = \frac{\ln(1 + \delta_{z})}{\ln(\kappa_{z})} \tag{94}$$

A6. Delta-Method Variance for $\hat{\theta}$

Let $\hat{\delta}_z$ be an estimator of δ_z with variance $\widehat{\mathrm{Var}}(\hat{\delta}_z)$ (from cluster-robust inference, bootstrap, or randomization inference inversion). Define $h(\delta) = \ln(1 + \delta)/\ln(\kappa_z)$. Then,

$$h'(\delta) = \frac{1}{\ln(\kappa_{z})} \cdot \frac{1}{1 + \delta}$$

The delta method gives

$$\widehat{\mathrm{Var}}(\hat{\theta}) \approx \left(\frac{1}{\ln(\kappa_{z})} \cdot \frac{1}{1 + \hat{\delta}_{z}}\right)^{2} \widehat{\mathrm{Var}}(\hat{\delta}_{z}) \tag{95}$$

Note*: These appendix formulas rely on the same undefined dose ratio $\kappa_z = S_z/S_0$. If $S_0 = 0$.

A7. Figures

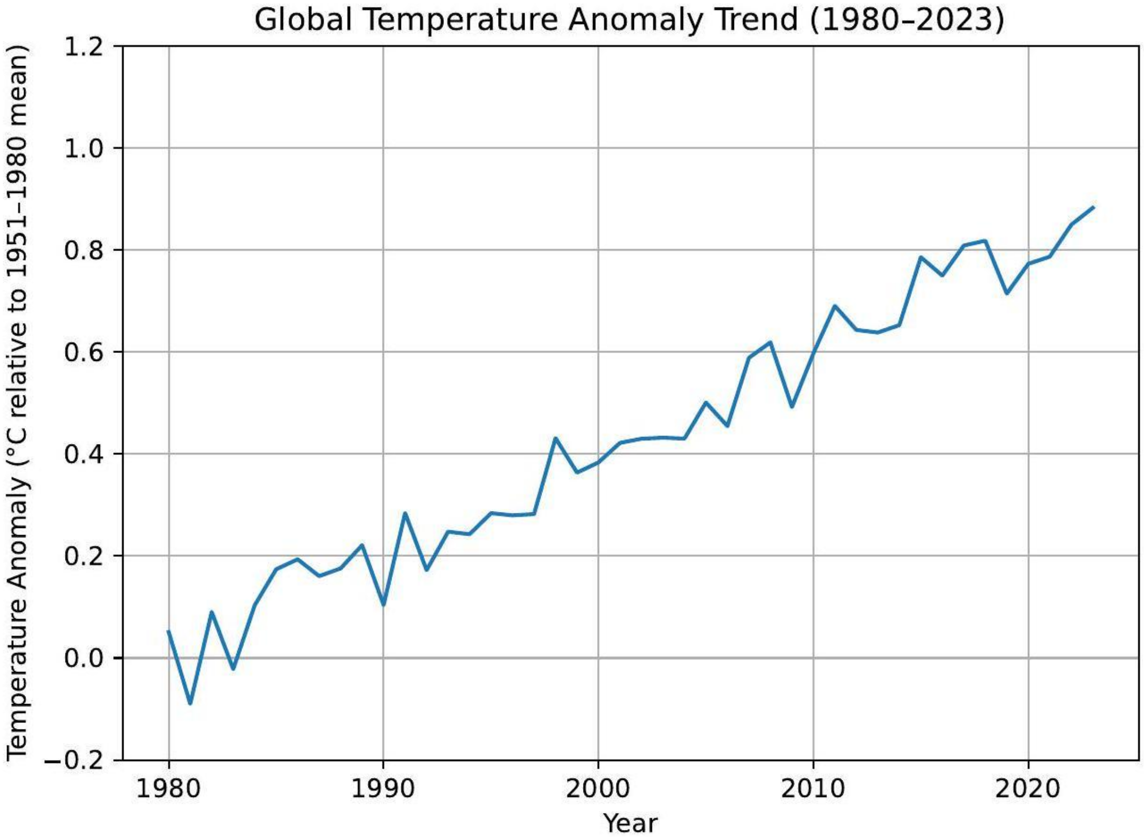

Figure 1: Global mean surface temperature anomaly (NASA GISTEMP, LandOcean). Annual global mean near-surface temperature anomalies in degrees Celsius relative to the NASA GISTEMP reference climatology (1951-1980 baseline as defined by the dataset), constructed from the J-D (Jan-Dec) column of GLB.Ts+dSST.csv. The series spans 1880-2025 with a partially reported 2026 row containing missing monthly values (marked *** in the raw file); the figure therefore reports complete annual means through 2025 and treats 2026 as incomplete. This time series also partially motivates the nonlinear yield-damage and risk environment assumed in the stochastic production model (Sections on stochastic production and farmer optimization), and it provides context for interpreting whether any estimated acoustic effects operate as the mean shifts, variance reductions, or both. Shortly: The plotted series is the global (LandOcean) annual anomaly; no additional smoothing is applied unless explicitly stated. Source file: GLB.Ts+dSST.csv (NASA GISTEMP global means table).

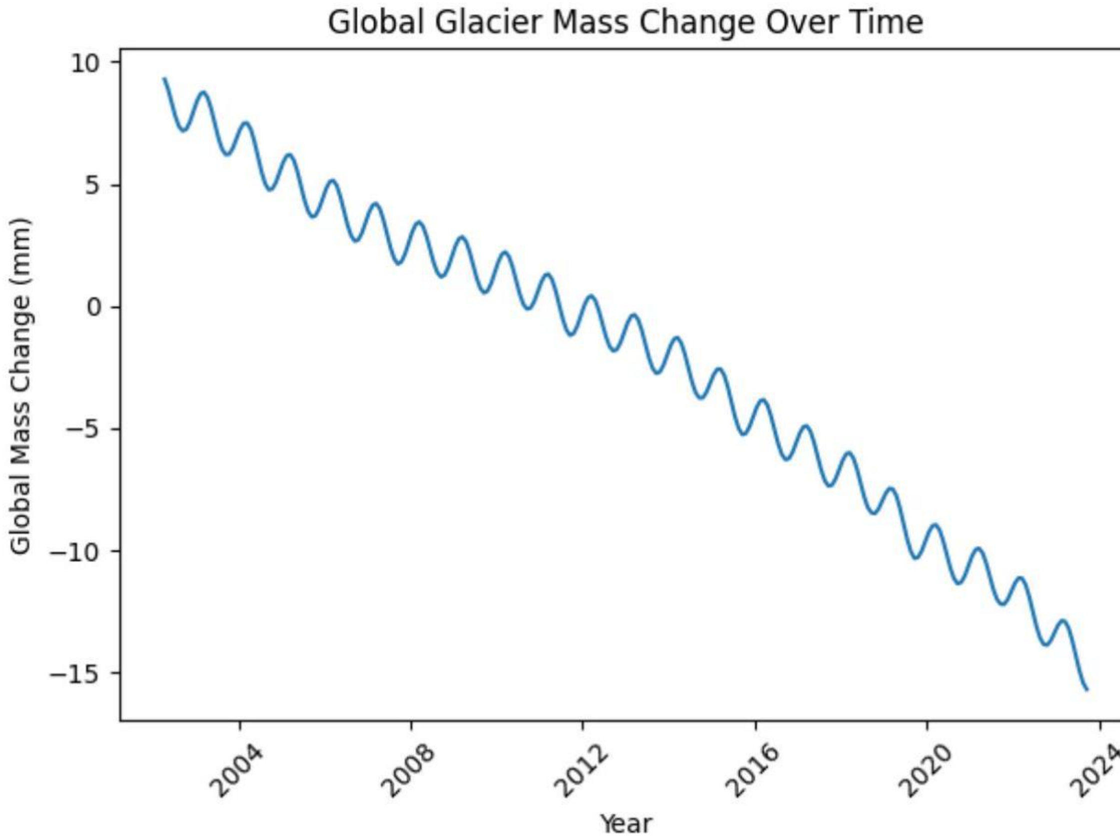

Figure 2: Global glacier mass loss. Time series of global glacier mass change (units as shown in the figure; commonly gigatons or meters water equivalent) over the displayed period. This figure is included as background evidence on long-run cryosphere decline, which is informative about persistent shifts in hydrological regimes and downstream agricultural water risk (linking to the groundwater and climate-risk motivation in the Introduction and the stochastic production environment).

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