

## Cosmochronology of White Dwarfs and Cooling Time Due to Heat Waves

Falcon Nelson<sup>1\*</sup> and Garcia Kelvin<sup>1</sup>

<sup>1</sup>Laboratory of Atmospheric and Outer Space Physics, FACYT, Department of Physics. University of Carabobo. Valencia, Carabobo State, Venezuela

**\*Corresponding Author:** Falcon Nelson, Laboratory of Atmospheric and Outer Space Physics, FACYT, Department of Physics. University of Carabobo. Valencia, Carabobo State, Venezuela.

**Citation:** Nelson, F. Kelvin, G. (2026). Cosmochronology of White Dwarfs and Cooling Time Due to Heat Waves. *Int J Astrophys Cosmol Space Explor.* 1(1), 01-11.

### Abstract

The standard white dwarf (WD) cooling theory is revised. The acausality implicit in the Maxwell-Fourier Law, used in the WD Standard Theory of Cooling, may be an invalid simplification in degenerate matter, where convective flows are correctly described by Cattaneo's Law. It is shown that the WD-cooling time is significantly shorter when considering causal propagation through heat waves in WD. The age of 7889 stars from the Gaia White Dwarf catalog is calculated. Additionally, the WD-age is estimated for very old stars, obtaining an average age of  $12.591_{-0.508}^{+0.544}$  Gyr for DA and  $12.589_{-0.728}^{+0.742}$  Gyr for DB. This cosmochronology allows to infer  $H_0 \cong 77.74_{-4}^{+4}$  km/sMpc in agreement with direct measurements of the Hubble constant.

**Keywords:** WD Cooling Time, WD: interior, Stellar Cosmochronology and Hubble tension

### Introduction: WD Radiative Transport

White dwarfs (WD) are the dense, hot remnant core left after the gravitational collapse of a low- or intermediate-mass star, when it exhausts all its nuclear fuel. Unable to perform further fusion in its core, the star ejects its outer layers (forming a planetary nebula) and the core contracts under the weight of its own gravity. These objects have effective surface temperatures ranging from 103 K to 105 K, and extraordinarily high densities, in the range of 104 to 107 g/cm<sup>3</sup> [1,2]. This high density is a consequence of a mass comparable to that of the Sun ( $M \sim M_{\odot}$ ) being confined in a volume approximately the size of the Earth ( $\sim 10-3R_{\odot}$ ). At these extremely high densities, electrons are stripped from their nuclei, creating an ultra-compact "sea" of nuclei floating on free electrons, and matter is said to be degenerate. The electrons' resistance to occupying the same quantum state generates a degeneracy pressure, independent of temperature, which is strong enough to counteract the force of gravity and halt the gravitational collapse. Lacking an internal power source, the white dwarf simply glows from trapped residual heat, over millions of years, it will slowly cool down.

If the energy is transported through the star layer by radiation, conduction and/or convection, the temperature gradient is given in the stellar interior in terms of opacity , density  $\rho$  and energy flux by the relation [2,3]: $\kappa F$

$$\frac{\partial T}{\partial r} = -\frac{3\kappa}{4ac} \frac{F}{T^3} \quad (1)$$

where  $a = 7,5710^{-15} \text{ erg.cm}^{-3} \text{ K}^{-4}$  is the radiation-density constant and  $c$  is the light velocity. Also, the hydrostatic equilibrium equation is:

$$\frac{\partial P}{\partial m} = -\frac{Gm}{4\pi r^4} \quad (2)$$

Using the Kramers-opacity  $\kappa = \kappa_0 P T^{-9/2}$ , the equations (1) and (2) can be integrated, then

$$T^{17/2} = \frac{51\kappa_0}{64\pi ac G M} \frac{L}{P^2} \quad (3)$$

The thermal conductivity of the electron gas is so high that the degenerate WD core is practically isothermal, whereas, conversely, heat transfer through the thin, non-degenerate surface layer is very slow and inefficient. The surface layer (the external convection zone) of the WD is crucial in determining its thermal structure and cooling times, which depend primarily on the internal temperature and are determined by the energy flow through the non-degenerate outer layers. Then using the ideal gas equations in the transition layer, we obtain the relationship between temperature, mass, and luminosity, normalized with respect to solar values [2]:

$$T^{7/2} = \vartheta \left( \frac{L/L_\odot}{M/M_\odot} \right) \quad (4)$$

Where the WD chemical composition is incorporated through the mass fractions of H, He and the metallicity by the constants  $X$ ,  $Y$  and  $B$  respectively.  $\mu$  is the average molecular weight,  $\mu_e$  is the average molecular weight per free electron

$$\vartheta = 5.790 \times 10^{25} [K^{7/2}] \left[ \frac{\mu_e^5 (1+X)(X+Y+B)}{\mu^4} \right] \quad (5)$$

Also, as the luminosity is associated with the temporal variation of internal energy, then:

$$-\frac{dL}{dm} = C_v \frac{dT}{dt} \rightarrow -L \approx \int_0^M C_v \frac{dT}{dt} dm \approx C_v M \frac{dT}{dt} \quad (6)$$

Where  $C_v$  is the specific heat. Thus, using (4) and (6), we obtain the differential equation for the cooling time

$$\frac{dT}{dt} = -\frac{L_\odot}{M_\odot} \frac{1}{C_v \vartheta} T^{7/2} \quad (7)$$

Integrating from  $t=0$  to the "lifetime" of the WD, we obtain standard cooling time for diffusion (diffusive cooling times) [2,3]:

$$t_v^{(d)} = \frac{2}{5} C_v \vartheta \left( \frac{M_\odot}{L_\odot} \right) T^{-5/2} \cong \frac{4.7 \times 10^7 [\text{yr}]}{A} \left( \frac{M/M_\odot}{L/L_\odot} \right)^{5/7} \quad (8)$$

with  $A$  the mass number.

This relationship has been widely used to constrain the age of white dwarfs in stellar cosmochronology and to compare it with the migration and breakup times of star clusters. The cooling time of white dwarfs is important for assessing the age of the galactic disk [4-6], as a standard chronometer for constraining the age of the Universe [7,8], and for describing the history of the Milky Way [9].

However, the preliminary calculation of the standard cooling time for heat diffusion begins by assuming the Maxwell-Fourier law for heat flux propagation (1), which is not valid for degenerate matter [10-12]. Consequently, we might ask how the cooling time changes if we use the causal generalization of the heat propagation law for luminosity (Cattaneo's law) and the implications of stellar cosmochronology for the age of the universe problem. To this end, we study the causal propagation of heat in degenerate material and derive the general equation for the cooling time of white dwarfs (section 2). Then, in section 3, we present the calculation of the age of white dwarfs, using heat waves, in the Gaia-DR2 catalog [13]. In the last two sections, we present the discussion about the Hubble tension and the general conclusions, respectively.

### **Methodology: Causal Heat Propagation**

We can see that (1) is just Fourier-Maxwell law for energy due to thermal conductivity in the approximation of diffusion:

$$\mathbf{F}(\mathbf{r}, t) = -k\nabla T(\mathbf{r}, t) \quad (9)$$

where the coefficient of conductivity for the diffusion energy.

$$k = \frac{4acT^3}{3\kappa} \quad (10)$$

It is well known that Fourier-Maxwell law leads to a parabolic equation for T, according to which perturbations propagate with infinite speed [10,11]. The origin of this non-causal behavior is found (7) which it is assumed that the energy flux appears at the same time the temperature gradient is switched on. A heat flux equation leading to a hyperbolic equation (telegraph equation) is the Cattaneo law:

$$\mathbf{F}(\mathbf{r}, t + \tau) = -k\nabla T(\mathbf{r}, t) \quad (11)$$

Where  $\tau$  is the relaxation time, this is the lapse that the flux of heat appears after a temperature gradient is switched on. Expanding by Taylor series the left side of (11) for small  $\tau$  we have

$$\tau \frac{\partial \mathbf{F}}{\partial t} + \mathbf{F} = -k\nabla T \quad (12)$$

Note that if the relaxation time is negligible, Cattaneo's law (12) is identical to Maxwell-Fourier's law (1). Neglecting the relaxation time is, in general, a sensible thing to do because for most materials it is very small of the order of 10-11 s for the phonon-electron

interaction and of the order 10-13 s for the phonon-phonon and free electron interaction. There are, however, situations where the relaxation time may not be negligible 10<sup>-3</sup> s for superfluid helium II and by the interior of a neutron star [10,11]. Thus, (12) can be written as:  $\tau_s$  for the phonon-phonon and free electron interaction. There are, however, situations where the relaxation time may not be negligible 10s for superfluid helium II and by the interior of a neutron star [10,11]. Thus, (12) can be written as:

$$\mathbf{F}(\mathbf{r}, t) = -\frac{k}{\tau} \int_{-\infty}^t \nabla T(\mathbf{r}, t') e^{-(t-t')/\tau} dt' \quad (13)$$

The temperature gradient can be approximated in terms of the temperature difference  $DT$ , between the surface and the core, of the white dwarf with radius  $R$ . Defining the heat flux  $\mathbf{F}$  as the product of the luminosity and the surface area, we obtain, using (5) that:

$$L = -C_v \frac{dDT}{dt} = \frac{4\pi Rk}{\tau} \int_{-\infty}^t DT(t') e^{-(t-t')/\tau} dt' \quad (14)$$

According to this result, the luminosity at an instant  $t$  contains the contributions of all previous thermal gradients; therefore, the luminous flux is greater at every instant and the white dwarf will cool more rapidly than in the diffusion approximation, which only responds to the instantaneous temperature gradient.

After using the Laplace transform, the differential equation can be integrated by  $DT$ , as:

$$DT(t) = DT(0) e^{-t/2\tau} \left[ \cos\left(\frac{\omega t}{2\tau}\right) + \frac{1-\omega^2}{2\omega} \sin\left(\frac{\omega t}{2\tau}\right) \right] \quad (15)$$

Where:

$$x = \frac{t}{2\tau}, \quad \omega^2 + 1 = \frac{4\tau}{\tau_d}, \quad \tau_d = \frac{\rho C_v R^2}{3k} = \frac{C_v M}{4\pi Rk} \quad (16)$$

Therefore, luminosity is expressed as:

$$L = L_0 e^{-t/\tau_d} e^{x(\omega^2-1)/2} \left[ \cos(\omega x) + \frac{\sin(\omega x)}{\omega} \right] \equiv L_0 e^{-t/\tau_d} \eta(x, \omega) \quad (17)$$

Note that the luminosity is not monotonous, but rather undergoes a damped oscillation as it propagates within the WD. Now, as before (section 1), substituting the true luminosity (17) into (6) and integrating from  $t=0$  to the "lifetime" of the WD, we obtain the cooling time when heat waves occur:

$$\int_0^{t_v} \frac{\eta(x, \omega)}{\left(1 - \frac{\tau}{\tau_d}\right) \eta(x, \omega) + \tau \frac{d\eta(x, \omega)}{dt}} dt = \frac{2}{5} C_v \vartheta \left(\frac{M_\odot}{L_\odot}\right) T_0^{-5/2} \quad (18)$$

Note that, when  $\tau \approx 0$ , we obtain the standard cooling equation (8). The general solution of (18) is:

$$\left(\frac{\tau_d}{2\tau}\right) t_v - \tau_d \ln \left| \cos\left(\frac{\omega t_v}{2\tau}\right) + \left(\frac{1-\omega^2}{2\omega}\right) \sin\left(\frac{\omega t_v}{2\tau}\right) \right| = t_v^{(d)} \quad (19)$$

Using (8) on the right-hand side, and the fact that the second term of the first member is bounded and very small, the preceding expression can be simplified as:

$$\left(\frac{\tau_d}{2\tau}\right) t_v \approx \frac{4.7 \times 10^7 [\text{yr}]}{A} \left(\frac{M/M_\odot}{L/L_\odot}\right)^{5/7} \quad (20)$$

Now, we calculate the thermal adjust time () used (16) with the heat capacity by degenerate material [10] and the thermal conductivity is dominated by electrons [10], then:  $\tau_d C_v \approx 10^{22} T (JK^{-2}) k \approx 10^{13} \rho T^{-1} (Wm^{-1}K^{-1})$

$$\tau_d \cong 80 \left[ \frac{T}{10^3 K} \right]^2 \left[ \frac{km}{R} \right] (s) \quad (21)$$

On other hand, the relaxation time () is given by relation [10], where  $\beta$  is the order of velocity of second sound in Helium[14,15], i.e . Then: $\tau \cong 10^{-3} \beta^{-2} \beta \sim c/\sqrt{3}$

$$t_v \approx \frac{2.45[Gyr]}{A} \left[ \frac{10^3 K}{T} \right]^4 \left[ \frac{R}{R_\odot} \right] \left[ \frac{M/M_\odot}{L/L_\odot} \right]^{5/7} \quad (22)$$

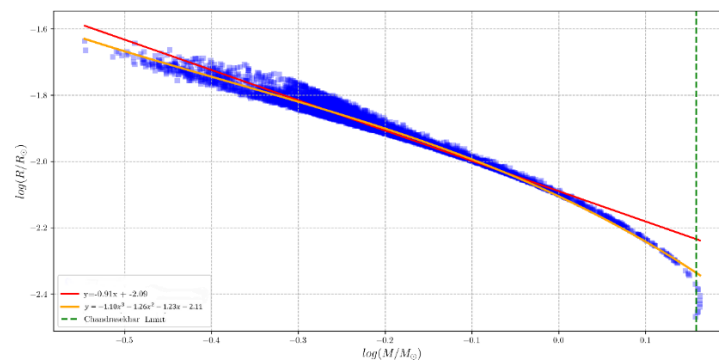
In the next section we will calculate the cooling time of the WD based on the astronomical observables.

## Results: Cooling Time and Heat Waves

The database used to apply (22) is the Gaia SVO white dwarf archive [16], which contains 73.221 white dwarfs identified using Gaia-DR2 and the Virtual Observatory (Jiménez-Esteban et al. 2018) [13]. This catalog was selected because it provides the necessary numerical values for mass, radius, and luminosity; with excellent astrometric and photometric measurements. It also shows other important values such as effective temperature ( $T_{eff}$ ), distance (dist.), and astronomical coordinates ( $RA$ ,  $DEC$ ). The data from this catalog were reduced to 7 887 confirmed white dwarf stars in the Galaxy with complete datasets.

Analysis of the mass-radius relationship (Fig. 1) reveals the characteristic behavior of objects sustained by electron degeneracy pressure, where there is an inverse correlation between these parameters. When modeling the distribution, the linear fit; i.e.  $\log(R/R_\odot) = 0.91\log(M/M_\odot)-2.09$ , allows us to identify a general trend of stellar contraction with increasing mass. Also, the third-degree polynomial fit allows us to show the asymptotic limit of the selected sample, as it approaches the Chandrasekhar limit when the slope of the distribution becomes steeper, i.e.:

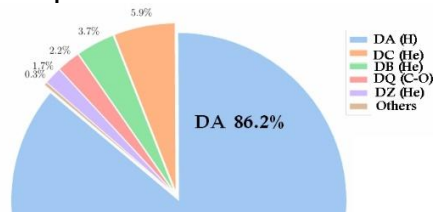
$$\log(R/R_\odot) = -1.10 \left( \log(M/M_\odot) \right)^3 - 1.26 \left( \log(M/M_\odot) \right)^2 - 1.23 \left( \log(M/M_\odot) \right) - 2.11. \quad (23)$$



**Figure 1: Radius-mass Ratio for the Selected Sample.**

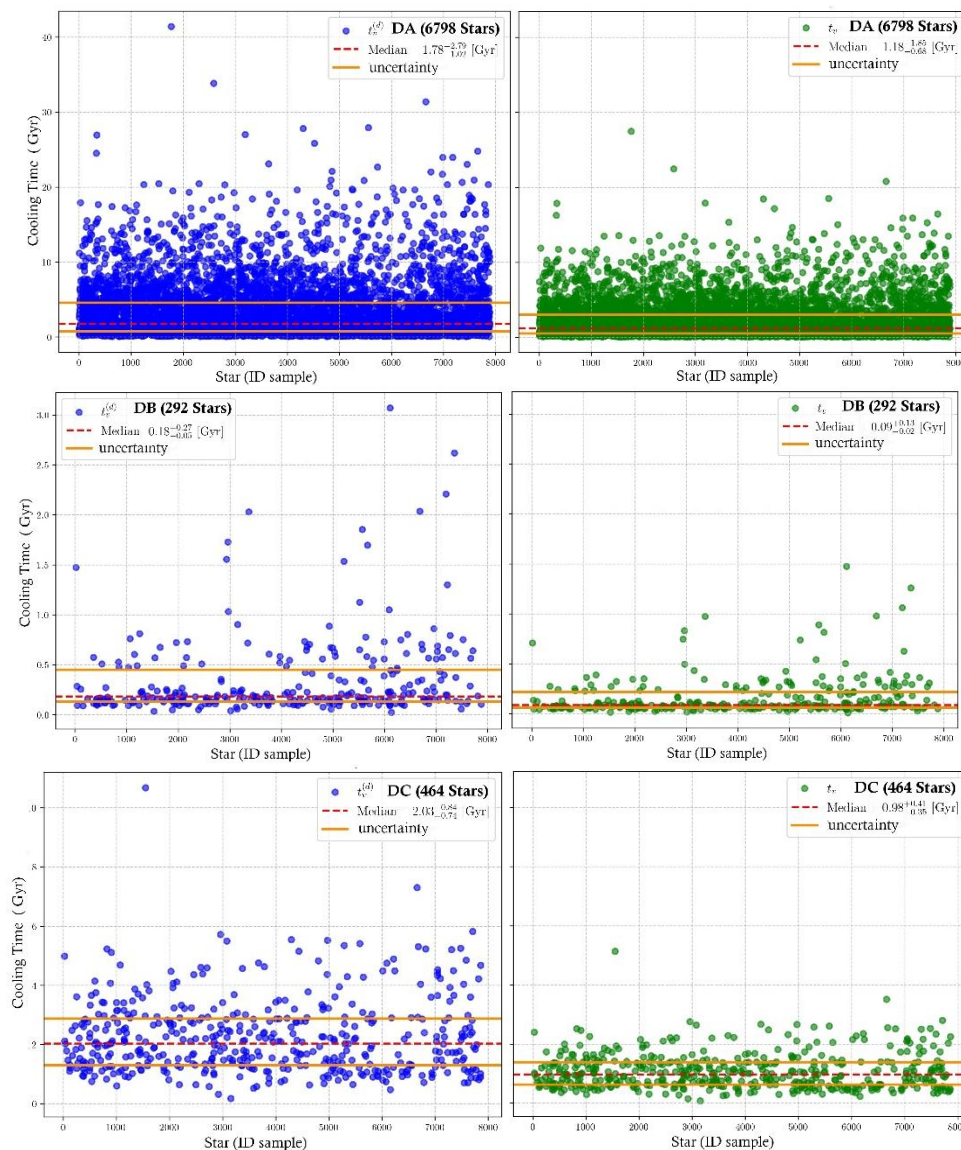
The Figure. 2 shows the percentage distribution of the white dwarf sample, classified according to their chemical composition. Hydrogen-rich white dwarfs (DA) predominate, constituting 86.2%. The remaining percentage is distributed among several helium-dominated subcategories: helium-type (DC) at 5.9% and helium-type (DB) at 3.7%. Stars with traces of carbon or oxygen (DQ, 2.2%) and those with metals or helium (DZ) (1.7%)

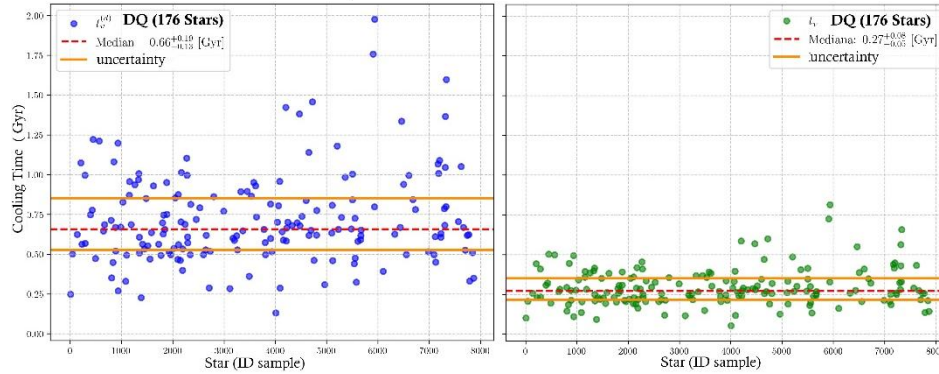
are present in smaller proportions, while other minor types with a defined composition represent only 0.3% of the sample.



**Figure 2: Spectral Classification of the WD in the Selected Sample (73221 Stars)**

Figure (3) illustrates the cooling time distribution for the studied sample, classified according to spectral type. The left panel presents the diffusive cooling times ( $t_v^{(d)}$ ) using Fourier's Law for heat propagation, and the right panel presents the cooling times ( $t_v$ ) calculated using Cattaneo's Law of causal heat propagation. It can be observed that, in general, heat propagation by waves significantly reduces the cooling time compared to the acausal approach.





**Figure 3: Comparison of Cooling Ages for the WD Sample by Spectral Type. Left Panel: Diffusive Cooling Times ( $t_v^{(d)}$ ) Based in Maxwell-Fourier Law, Right Panel: Causal Cooling Times ( $t_v$ ), Using Cattaneo's Law (heat Waves).**

Analysis of the majority population of hydrogen-atmospheric white dwarfs (DAs) reveals a marked acceleration in the cooling process. While classical models predict a median age of  $1.78^{+2.79}_{-1.02}$  Gyr, equation (22) yields a value of  $1.18^{+1.85}_{-0.68}$  Gyr. This reduction suggests that galactic chronology based on these objects may require recalibration; star clusters and galactic disk structures dated using these stars could be substantially younger than currently accepted.

In the case of neutral helium-dominated stars (DB type), considered among the earliest cooling phases after the Asymptotic Giant Branch (AGB), the diffusive cooling times  $t_v^{(d)} \simeq 0.18^{+0.27}_{-0.05}$  Gyr. In contrast, the "true" cooling time, that is, through heat waves, reduces this transition to  $t_v \simeq 0.09^{+0.13}_{-0.02}$  Gyr. The halving of this period suggests that the initial transition of stars after the AGB phase, as well as the gravitational collapse of residual hydrogen in the atmosphere, occurs at a highly accelerated rate, pointing to rapid energy dissipation processes via initial heat waves, which are omitted in the standard formalism based on the acausal propagation of heat. This would also explain why there are fewer DB stars compared to the detected population of DA stars.

Continuous-spectrum (DC) white dwarfs, associated with advanced and cool stages, show a discrepancy that remains constant over the long term. The diffusive cooling times estimate is  $t_v^{(d)} \simeq 2.03^{+0.84}_{-0.74}$  Gyr, compared to cooling times  $t_v \simeq 0.98^{+0.41}_{-0.35}$  Gyr calculated with our model. This drastic difference in the more mature stages implies that mechanisms such as core crystallization or deep convective coupling extract thermal energy less efficiently than mechanisms based on wave-like heat propagation, directly affecting the dating of the oldest stellar populations in the galactic halo.

For the subsample of DQ-type white dwarfs, the discrepancy is even more pronounced. The acausal approach projects a median of  $0.66^{+0.19}_{-0.13}$  Gyr, but our model determines only  $t_v \simeq 0.27^{+0.08}_{-0.05}$ . This difference implies that the internal mixing and convective dredging mechanisms that transport carbon from the core to the outer layers operate with a much

higher thermal efficiency (heat waves), developing on much shorter evolutionary timescales than those stipulated by theoretical models calculated using the Maxwell-Fourier La

### Discussion: Stellar Cosmochronology

The age of the progenitor stars can be estimated from the masses of the WD remnants known. [17,18], then:

$$\begin{aligned} M_{WD} &\cong 0.08 \cdot M_{prog} + 0.49 & 0.83M_{\odot} < M_{prog} < 2.85M_{\odot} \\ M_{WD} &\cong 0.19 \cdot M_{prog} + 0.18 & 2.85M_{\odot} < M_{prog} < 3.60M_{\odot} \end{aligned} \quad (24)$$

Also, knowing the mass of the progenitor star, the time spent on the main sequence, including the red giant phase, can be calculated through the relation [19]:

$$t_{MS} = \left( \frac{M_{prog}}{M_{\odot}} \right)^{-2.9} [10 \text{ Gyr}] \quad (25)$$

Therefore, the total age of a WD will be the sum of the cooling time plus the time spent in the nuclear phase ( $t_{MS}$ ). These correspond to the mean values of all 7889 stars. The overall results are shown in Table 1. The second and third columns correspond to the acausal diffusive approximation, and the last columns correspond to the calculation corrected by Cattaeno's Law for degenerate matter and cooling by heat waves.

**Table 1: Mean Values Cooling Time and WD-age**

<b>WD Spectral Type</b>	Sample #stars/7889 [16]	$t_v^{(d)}$ [Gyr] (difussive) <b>Eq(8)</b>	$t_T^{(d)}$ [Gyr] (difussive) $t_v^{(d)} + t_{MS}$	$t_v$ [Myr] (Heat Waves) <b>Eq (22)</b>	$t_T$ [Gyr] (Heat Waves) $t_v + t_{MS}$
DA	6798 (86.19%)	$1.781^{+2.786}_{-1.018}$	$5.852^{+3.479}_{-2.467}$	$7.804^{+27.270}_{-5.765}$	$2.326^{+2.391}_{-1.689}$
DB	292 (3.70%)	$0.183^{+0.268}_{-0.052}$	$1.894^{+2.252}_{-0.909}$	$0.432^{+1.293}_{-0.142}$	$1.399^{+2.543}_{-0.925}$
DC	464 (5.88%)	$2.029^{+0.843}_{-0.736}$	$2.385^{+0.899}_{-0.789}$	$14.220^{+11.190}_{-7.603}$	$0.183^{+0.104}_{-0.609}$
DQ	176 (2.23%)	$0.658^{+0.194}_{-0.131}$	$0.912^{+0.239}_{-0.131}$	$4.924^{+2.672}_{-1.554}$	$0.194^{+0.123}_{-0.054}$
DZ	133 (1.69%)	$1.728^{+0.465}_{-0.469}$	$1.871^{+0.556}_{-0.391}$	$9.977^{+5.021}_{-3.981}$	$0.161^{+0.071}_{-0.034}$

The mean values for ages and cooling times in Table 1 must be cleaned because the selected sample includes WDs of very different masses and radii (Fig. 1). Filtering the sample only for the oldest stars, whose masses and luminosities are in the range  $0.563^{+0.015}_{-0.015} M_{\odot}$   $0.0158^{+0.025}_{-0.025} L_{\odot}$  respectively, we obtain Table 2. The average age of this selected subsample is observed to be on the order of 12 billion years, and consequently, the Hubble constant is at least 73.7 km/s Mpc. The age of these older WDA-WDBs is in agreement with the values reported  $12.5^{+1.4}_{-3.4}$  Gyr., for WD in the inner halo of Milky Way, by Kilit et al. [5].

**Table 2: Cooling Time and WD-age for Old**

WD Spectral Type	Sample #stars/7889	$t_T^{(d)}$ [Gyr]	$H_0 \left[ \frac{km}{Mpc \cdot s} \right]$	$t_T$ [Gyr] (Heat Waves)	$H_0 \left[ \frac{km}{Mpc \cdot s} \right]$
DA	217/6798 (3.19%)	$12.719^{+0.072}_{-0.778}$	$76.933^{+5.082}_{-4.023}$	$12.591^{+0.544}_{-0.508}$	$77.715^{+3.268}_{-3.220}$
DB	12/292 (4.11%)	$12.744^{+0.645}_{-0.931}$	$76.780^{+6.050}_{-3.701}$	$12.589^{+0.742}_{-0.728}$	$77.727^{+4.777}_{-4.326}$
DC	4/464 (0.86%)	$13.162^{+0.290}_{-0.063}$	$74.345^{+0.357}_{-1.567}$	$12.409^{+0.107}_{-0.206}$	$78.855^{+1.357}_{-0.669}$
Mean		<b>12.728</b>	<b>76.8806</b>	<b>12.588</b>	<b>77.7352</b>

The small sample size used in Table 2 (second column) suggests a need for further refinement of the analysis to establish the age of the Milky Way's inner halo without potential statistical bias. Other estimates of the age of the WD in the Milky Way's halo have reported Gyr [6], 10 Gyr [4]. 11.42 Gyr [20] This would be a subsequent endeavor beyond the scope of this work, whose objective is to demonstrate that the cooling time of the Milky Way's WD is shorter than assumed when considering causal heat propagation, using the Cattaneo equation.11.2

Another important aspect is the Hubble Constant Hubble ( ) inferred in Table 2 (last column), which is consistent with a higher value that inferred from the Planck Satellite data ( ), indeed [21], [22], [23] and [24]- $H_0 = 77.7 \text{ km/sMpc}$   $H_0 \approx 67.74 \text{ km/sMpc}$   $H_0 = 75.8^{+5.2}_{-4.9} \text{ km/sMpc}$   $H_0 = 74.22^{+1.8}_{-1.8} \text{ km/sMpc}$   $H_0 = 72.14^{+2.4}_{-2.4} \text{ km/sMpc}$   $H_0 \leq 86.6 \text{ km/sMpc}$

### Summary and Conclusions

The standard cooling time for heat diffusion is based on the assumption of the Maxwell-Fourier law for heat flux propagation (1), which is not valid for degenerate matter. The Fourier-Maxwell law leads to a parabolic equation for T, according to which perturbations propagate at infinite speed. Consequently, the cooling time changes if we use the causal generalization of the law of causal heat propagation in luminosity, Cattaneo's Law (13). This heat flux equation leads to a hyperbolic equation where the relaxation time may not be negligible (for superfluid helium II and within a compact star). Therefore, luminosity is expressed as a damped oscillation as it propagates within the white dwarf, by means of heat waves. Then WD-cooling time is significantly shorter (see Eq 22 and Fig 3).

The GAIA-DR2 catalog of 7887 confirmed white dwarfs was used. The analysis of the mass-radius relationship (Figure. 1) reveals the characteristic behavior of objects sustained by electron degeneracy pressure. And that of the majority population of hydrogen-atmospheric white dwarfs reveals a marked acceleration in the cooling process. This reduction suggests that galactic chronology based on these objects may require recalibration; star clusters and galactic disk structures dated with these stars could be substantially younger than currently accepted, raising implications for stellar cosmochronology regarding the age of the universe. Furthermore, the age of white dwarfs

is estimated for very old stars, yielding an average age of  $12.591_{-0.508}^{+0.544}$  Gyr for DA and  $12.589_{-0.728}^{+0.742}$  Gyr for DB. This cosmochronology allows us to infer  $H_0 \cong 77.74_{-4}^{+4}$  km/sMpc in accordance with direct measurements of the Hubble constant.

## References

- [1] Van Horn, H. M.: 1984, The physics of white dwarfs, in *Astrophysics Today*, Cameron, A.G.W. editor, American Institute of Physics, N. Y.
- [2] Kippenhahn, R., Weigert, A., and Weiss, A. (2012). *Stellar structure and evolution*. Berlin: Springer-verlag.
- [3] Hansen, B. M. (1999). Cooling models for old white dwarfs. *Astrophysical Journal*, 520(2): 680-695.
- [4] Tremblay, P. E. et al (2014). White dwarf cosmochronology in the solar neighborhood. *Astrophysical Journal*, 791(2), 92.
- [5] Kilic, M et al. (2017) The Ages of the Thin Disk, Thick Disk, and the Halo from Nearby White Dwarfs, *Astrophysical Journal* 873: 162-171
- [6] Krauss, L. M., and Chaboyer, B. (2003). Age estimates of globular clusters in the Milky Way: constraints on cosmology. *Science*, 299(5603), 65-69.
- [7] Winget, D. E. et al (1987). An independent method for determining the age of the universe. *Astrophysical Journal Letters* 315: L77-L81.
- [8] Saumon, D., Blouin, S., and Tremblay, P. E. (2022). Current challenges in the physics of white dwarf stars. *Physics Reports*, 988, 1-63.
- [9] Fantin, N. J. et al (2019). The Canada–France Imaging Survey: reconstructing the Milky Way star formation history from its white dwarf population. *Astrophysical Journal*, 887(2): 148.
- [10] Herrera, L. and Falcon, N. (1995) Heat waves and thermohaline instability in a fluid. *Physics Letters A* 201: 33-37.
- [11] Falcon, N. (2001) Compact star cooling by means of heat waves, *RMAA*, 11: 41-42.
- [12] Falcon, N. (2005, July). White Dwarf Cooling by Means of Heat Waves. In 14th European Workshop on White Dwarfs, ASP Conferences Series 334, 69-72.
- [13] Jiménez-Esteban, F. et al (2018). A white dwarf catalogue from Gaia-DR2 and the Virtual Observatory. *MNRAS* 480(4), 4505-4518.
- [14] Falcon, N. (2003). Heat waves and WD cooling theory in ZZ Ceti stars. In D. de Martino, R. Silvotti, J.-E. Solheim y R. Kalytis (Eds.), *White Dwarfs*, NATO 105: 223–224. Kluwer Academic Publishers
- [15] Falcon, N. and Labrador, J., (2001) Thermal Waves and Unstable Convection in ZZ Ceti Stars, *Odessa Astronomical Publications* 14: 141–143.
- [16] [The Gaia White Dwarf SVO Archive \(2026\)](#).
- [17] Cummings, J. D. et al (2018). The white dwarf initial–final mass relation for progenitor stars from 0.85 to 7.5  $M_{\odot}$ . *Astrophysical Journal*, 866(1), 21.
- [18] Salaris, M., Serenelli, A., Weiss, A., Bertolami, M. M. (2009). Semi-empirical white dwarf initial–final mass relationships: a thorough analysis of systematic uncertainties due to stellar evolution models. *The Astrophysical Journal*, 692(2), 1013-1032.
- [19] Hansen, C. J., Kawaler, S. D., Trimble, V. (2004). *Stellar interiors: physical principles, structure and evolution*. 2nd ed. A&A library. Springer-Verlag New York

- [20]Falcon, N. (2021). A large-scale heuristic modification of Newtonian gravity as an alternative approach to dark energy and dark matter. *Journal of Astrophysics and Astronomy*, 42(2), 102.
- [21]De Jaeger, T et al (2020). A measurement of the Hubble constant from Type II supernovae. *Monthly Notices of the Royal Astronomical Society*, 496(3), 3402-3411.
- [22]Riess, A. G. et al (2019). Large Magellanic Cloud Cepheid standards provide a 1% foundation for the determination of the Hubble constant and stronger evidence for physics beyond  $\Lambda$ CDM. *The Astrophysical Journal*, 876(1), 85.
- [23]Falcon, N., Aguirre A. (2014). Theoretical Deduction of the Hubble Law Beginning with a MoND Theory in Context of the  $\Lambda$ FRW-Cosmology. *International Journal of Astronomy and Astrophysics*. 4. 551-559
- [24]Falcon, N. (2025) Zwicky's Missing Mass : Dark Matter Versus Modified Gravity.